Journal of Applied Mechanics and Technical Physics, Vol. 50, No. 5, pp. 850–857, 2009

DIRECT AND INVERSE PROBLEMS OF OSCILLATIONS OF AN ELASTOLIQUID LAYERED MEDIUM

P. S. Uglich

UDC 534.26

A plane problem of forced oscillations of an ideal compressible liquid bounded from above by an elastic layer with a rough lower surface and an inverse geometric problem of determining the shape of the rough lower surface of an elastic layer from the wave characteristics on the upper surface are considered. Three methods are used to solve the direct problem: the small parameter method, the boundary element method, and the Born approximation. Solving the inverse problem is reduced to solving the integral Fredholm equation of the first kind. Results of a numerical experiment are presented.

Key words: *elasticity theory, acoustics, boundary element method, inverse problems, ill-posed problems.*

Introduction. Problems with vibrations of layered media containing defects (such as cavities, cracks, or inclusions) have been considered rather frequently. Problems of this kind are solved by the method of boundary integral equations, based on reducing the initial problem to a system of integral equations. The resultant system, in turn, is reduced to a well-posed system of linear algebraic equations, which can be easily solved. The method of constructing such equations is described in detail, for instance in [1].

Inverse problems of determining the character and shape of the defect on the basis of the character of the wave field on the surface of the layered medium are also of interest. In particular, such problems were solved in [2, 3] by the method of linearization under the assumption that the roughness amplitude is small. Direct and inverse problems of wave scattering on small compact inhomogeneities in a sea waveguide were considered in [4, 5]; in that study, the roughness amplitude was assumed to be small, as compared with the wave length, and the roughness itself was specified on a bounded set. The Born approximation was used to solve the direct problem in [6].

Vibrations of an ideal compressible liquid bounded from above by an elastic layer with a rough lower surface are considered in the present work. A normal load is applied to the upper surface of the layer, the condition of no penetration of the liquid is imposed on the lower surface, the shear stress is absent, and the normal stress is equal to the pressure in the liquid.

Three methods are used to solve the problem posed. The first method is based on the method of boundary integral equations and allows the initial problem to be reduced to a system of singular integral equations, which is solved numerically by the boundary element method. The second method is based on the Born approximation and allows the field of displacements on the upper surface of the layer to be found without solving the system of boundary integral equations. The third method is based on the perturbation method and allows the initial boundary-value problem to be reduced to a sequence of inhomogeneous boundary-value problems for a smooth layer. The latter approach is applicable in the case of a small roughness amplitude.

In the present work, we formulate an inverse problem on finding the shape of a rough area of the lower surface from the known field of displacement on the upper surface of the layer. Under the assumption of a small depth of the roughness, the problem reduces to solving the integral Fredholm equation of the first kind with a smooth

South Federal University, Rostov-on-Don 344006; puglich@inbox.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 50, No. 5, pp. 143–151, September–October, 2009. Original article submitted October 6, 2008.

kernel. The solution of such an equation is an ill-posed problem and is constructed by Tikhonov's regularization method [7].

1. Formulation of the Problem. Let us consider a plane formulation of the problem of steady oscillations of a semi-infinite layered medium ($|x_1| < \infty$ and $x_2 \le 1$) consisting of an elastic layer $|x_1| < \infty$, $0 \le \varepsilon f(x_1) \le x_2 \le 1$ with a rough lower surface, which contacts an ideal compressible liquid occupying the domain $|x_1| < \infty$, $x_2 < \varepsilon f(x_1)$, where ε is a small parameter and $f(x_1)$ is a rather smooth function that describes the roughness shape [$f(x_1) \ne 0$ at $x_1 \in [a, b]$ and $f(a) = f(b) = 0$].

Steady oscillations are described by the equations

$$(1-2\nu)^{-1} u_{j,ji} + \Delta u_i + \varkappa_2^2 u_i = 0,$$
$$\Delta\varphi + \varkappa_0^2 \varphi = 0, \tag{1}$$

where φ is the velocity potential in the liquid, $\varkappa_2 = \omega/c_2$, $\varkappa_0 = \omega/c_0$, ω is the frequency of oscillations, $c_2 = \sqrt{G/\rho}$ is the velocity of transverse waves in the elastic layer, c_0 is the velocity of sound in the liquid, G is the shear modulus, and ν is Poisson's ratio for the layer.

We assume that a normal load is applied to the upper surface of the layer and there are no shear stresses:

$$\sigma_{12}\Big|_{x_2=h} = 0, \qquad \sigma_{22}\Big|_{x_2=h} = q(x_1). \tag{2}$$

In what follows, the layer thickness h is assumed to be equal to unity without loss of generality.

The tangential components of the stress vector on the lower surface of the layer are equal to zero, and the normal component is equal to the pressure in the liquid. At $x_2 = \varepsilon f(x_1)$, in addition, the interface between the layer and the liquid obeys the conjugation conditions

$$\sigma_{n\tau} = 0, \qquad \sigma_{nn} = -i\omega\rho_*\varphi, \qquad -i\omega u_n = \frac{\partial\varphi}{\partial n}, \tag{3}$$

where $\boldsymbol{n}$ and $\boldsymbol{\tau}$ are the normal and tangential vectors to the curve $x_2 = \varepsilon f(x_1)$, and ρ_* is the liquid density. The problem formulation is closed by the conditions of radiation at infinity, which are formulated on the basis of the ultimate absorption principle [8].

2. Derivation of the System of Boundary Integral Equations. Let $U_i^{(m)}$ be the fundamental solution for the layer–liquid system (1) with straight-line boundaries, which satisfies the following equations and boundary conditions:

$$(1-2\nu)^{-1} U_{k,ki}^{(m)} + \Delta U_i^{(m)} + \varkappa_2^2 U_i^{(m)} + \frac{\delta_{im}}{G}\,\delta(x-\xi) = 0;$$
$$\Delta\Phi^{(m)} + \varkappa_0^2 \Phi^{(m)} = 0; \tag{4}$$

$$G(U_{1,2}^{(m)} + U_{2,1}^{(m)})\Big|_{x_2=0,1} = 0,$$
$$\frac{2G}{1-2\nu}\Big[\nu U_{1,1}^{(m)} + (1-\nu) U_{2,2}^{(m)}\Big]\Big|_{x_2=1} = 0,$$
$$\frac{2G}{1-2\nu}\Big[\nu U_{1,1}^{(m)} + (1-\nu) U_{2,2}^{(m)} + \frac{i\rho_*\omega}{G}\,\Phi^{(m)}\Big]\Big|_{x_2=0} = 0, \tag{5}$$
$$-i\omega U_2^{(m)}\Big|_{x_2=0} = \Phi_{,2}^{(m)}\Big|_{x_2=0}.$$

Let us use $U_i^-(x,\xi)$ to indicate the fundamental solution for the layer–liquid system in the case with the source of disturbances located in the liquid:

$$(1-2\nu)^{-1} U_{k,ki}^- + \Delta U_i^- + \varkappa_2^2 U_i^- = 0,$$
$$\Delta\Phi^- + \varkappa_0^2 \Phi^- = -\delta(x-\xi).$$

The boundary conditions for the functions U_i^- and Φ^- have the form (5). Let us denote the area occupied by the layer by S_+, the upper surface of the layer by L_+, the lower surface of the layer by L_-, and the rough area of the boundary by Γ_f.

Let us consider the domain $S_R^+ = S_+ \cap \{|x_1| < R\}$. Using the Somigliana formula, we obtain

$$u_m(\xi) = \int\limits_{\partial S_R^+} \Big[\sigma_{kl}(x) n_l U_k^{(m)}(x,\xi) - u_k(x)\sigma_{kl}^{(m)}(x,\xi) n_l\Big]\, dl_x, \qquad \sigma_{nn} = \sigma_{ij} n_i n_j.$$

Directing R to infinity and taking into account the boundary conditions, we obtain

$$u_m(\xi) = u_m^{et}(\xi) + \int\limits_{L_-} \Big[\sigma_{nn}(x) U_n^{(m)}(x,\xi) - u_k(x)\sigma_{kl}^{(m)}(x,\xi) n_l\Big]\, dl_x, \tag{6}$$

where $u_k^{et}(\xi)$ is the field of displacements in the layer with straight-line boundaries under the action of the load $q(x_1)$.

Let us consider the domain S_R^- bounded from above by the curve L_- and from below by a semicircle of radius R with the center in the origin. We multiply the second equation of system (1) by the function $\Phi^{(m)}$ and Eq. (4) by the function φ, subtract the resultant equations one from the other, and integrate over the domain S_R^-:

$$\int\limits_{S_R^-} \Big[\Delta_x \Phi^{(m)}(x,\xi)\varphi(x) - \Delta_x\varphi(x)\Phi^{(m)}(x,\xi)\Big]\, dS_x = \int\limits_{\partial S_R^-} \Big(\frac{\partial \Phi^{(m)}(x,\xi)}{\partial n}\,\varphi(x) - \frac{\partial\varphi(x)}{\partial n}\,\Phi^{(m)}(x,\xi)\Big)\, dl_x.$$

Directing R to infinity and taking into account the boundary conditions on the layer–liquid interface, we obtain

$$-i\int\limits_{\Gamma_f} \Big[\frac{\partial\Phi^{(m)}(x,\xi)}{\partial n}\Big(\frac{\sigma_{nn}(x)}{\rho_*\omega}\Big) - \omega u_n(x)\Phi^{(m)}(x,\xi)\Big]\, dl_x + \frac{1}{\rho_*}\int\limits_{L_-\setminus\Gamma_f} \Big[\sigma_{22}(x) U_2^{(m)}(x,\xi) - u_2(x)\sigma_{22}^{(m)}(x,\xi)\Big]\, dl_x = 0. \tag{7}$$

Multiplying Eq. (7) by ρ_* and subtracting it from Eq. (6), we obtain

$$u_m(\xi) = u_m^{et}(y) + \int\limits_{\Gamma_f} \Big[\sigma_{nn}(x)\Big(U_n^{(m)}(x,\xi) + \frac{i}{\omega}\,\frac{\partial\Phi^{(m)}(x,y)}{\partial n}\Big)$$

$$- u_n(x)\Big[\sigma_{nn}^{(m)}(x,\xi) + i\omega\rho_*\Phi^{(m)}(x,\xi)\Big] - u_\tau(x)\sigma_{n\tau}^{(m)}(x,\xi)\Big]\, dl_x. \tag{8}$$

With known functions u_n, u_τ, and σ_{nn}, Eq. (6) allows us to determine the field of displacements in the layer at an arbitrary point ξ satisfying the condition $\xi > \varepsilon \max f(x_1)$. Let us consider the half-plane $x_2 < 0$ for which we can find a relation of the form

$$\int\limits_{-\infty}^{\infty} \Big(\frac{\partial\Phi^{(m)}(x,\xi)}{\partial n}\,\varphi(x) - \frac{\partial\varphi(x)}{\partial n}\,\Phi^{(m)}(x,\xi)\Big)\, dl_x = 0. \tag{9}$$

We multiply Eq. (9) by ρ_* and subtract it from Eq. (6). Taking into account the boundary conditions on the layer–liquid interface, we obtain

$$u_m(\xi) = u_m^{et}(\xi) - \int\limits_{\Gamma_f} \Big[i\omega\rho_*\varphi(x) U_n^{(m)}(x,\xi) + u_k(x)\sigma_{kl}^{(m)}(x,\xi)\Big]\, dl_x$$

$$+ \rho_*\int\limits_a^b \Big[\psi(x_1)\Phi_{,2}^{(m)}(x_1,0;\xi) - \chi(x_1)\Phi^{(m)}(x_1,0;\xi)\Big]\, dx_1. \tag{10}$$

As $\xi \to y \in \Gamma_f$, we obtain

$$\frac{1}{2}\, u_m(y) = u_m^{et}(y) - \int\limits_{\Gamma_f} \Big[i\omega\rho_*\varphi(x) U_n^{(m)}(x,y) + u_k(x)\sigma_{kl}^{(m)}(x,y)\Big]\, dl_x$$

$$+ \rho_*\int\limits_a^b \Big[\psi(x_1)\Phi_{,2}^{(m)}(x_1,0;y) - \chi(x_1)\Phi^{(m)}(x_1,0;y)\Big]\, dx_1. \tag{11}$$

The following notation is used in Eqs. (10) and (11):

$$\psi(x_1) = \varphi(x)\Big|_{x_2=0}, \qquad \chi(x_1) = \varphi_{,2}(x)\Big|_{x_2=0}.$$

Let us consider the domain $\{a \le x_1 \le b,\ \ 0 \le x_2 \le \varepsilon f(x_1)\}$. Using the Green formula for the Helmholtz equation with $\xi \to y \in \Gamma_f$, we obtain

$$\frac{1}{2}\,\varphi(y) = -\int\limits_{\Gamma_f} \Big(i\omega\rho_* u_n(x)\Phi^0(x,y) + \varphi(x)\,\frac{\partial\Phi^0}{\partial n}(x,y)\Big)\,dl_x - \int\limits_a^b \Big[\psi(x_1)\Phi^0_{,2}(x_1,0;y) - \chi(x_1)\Phi^0(x_1,0;y)\Big]\,dx_1. \quad (12)$$

Directing the point ξ to the straight line $x_2 = 0$, we obtain an equation on the segment $[a,b]$ of the form

$$\frac{1}{2}\,\psi(y) = -\int\limits_{\Gamma_f} \Big(i\omega\rho_* u_n(x)\Phi^0(x;y_1,0) + \varphi(x)\,\frac{\partial\Phi^0}{\partial n}(x;y_1,0)\Big)\,dl_x$$
$$- \int\limits_a^b \Big[\psi(x_1)\Phi^0_{,2}(x_1,0;y_1,0) - \chi(x_1)\Phi^0(x_1,0;y_1,0)\Big]\,dx_1. \quad (13)$$

The following relation is valid in the domain $x_2 \le 0$:

$$-\frac{1}{2}\,\psi(y_1,0) = Q(y_1) - \frac{1}{\rho_*}\int\limits_{\Gamma_f} \Big[i\omega\rho_*\varphi(x)U_n^-(x;y_1,0) + u_k(x)\sigma_{kl}^-(x;y_1,0)\Big]\,dl_x$$
$$- \int\limits_a^b \Big[\psi(x_1)\Phi^-_{,2}(x_1,0;y_1,0) - \chi(x_1)\Phi^-(x_1,0;y_1,0)\Big]\,dx_1, \quad (14)$$

$$Q(y_1) = \int\limits_{-\infty}^{\infty} q(x_1)U_2^-(x_1,1;y_1,0)\,dx_1.$$

Thus, we obtain a system of five integral equations (11)–(14) with respect to five unknown functions.

3. Perturbation Method. Assuming the depth to be small, we obtain the following equation for the stress tensor components on the rough boundary:

$$\sigma_{nn} = \sigma_{22} - 2\varepsilon f'(x_1)\sigma_{12} + O(\varepsilon^2), \qquad \sigma_{n\tau} = -\sigma_{12} - \varepsilon f'(x_1)(\sigma_{22} - \sigma_{11}) + O(\varepsilon^2). \quad (15)$$

We introduce perturbations for u_n and $\partial\varphi/\partial n$ in a similar manner:

$$u_n = -u_2 + \varepsilon f'(x_1)u_1 + O(\varepsilon^2), \qquad \frac{\partial\varphi}{\partial n} = -\varphi_{,2} + \varepsilon f'(x_1)\varphi_{,1} + O(\varepsilon^2). \quad (16)$$

At $x_2 = \varepsilon f(x_1)$, we expand the boundary values of the functions $u_i(x_1,x_2)$ and $\varphi(x_1,x_2)$ into the Taylor series with respect to x_2 in the neighborhood of the point $x_2 = 0$:

$$u_i(x_1,\varepsilon f(x_1)) = u_i(x_1,0) + \varepsilon f(x_1)u_{2,i}(x_1,0) + O(\varepsilon^2),$$

$$\varphi(x_1,\varepsilon f(x_1)) = \varphi(x_1,0) + \varepsilon f(x_1)\varphi_{,2}(x_1,0) + O(\varepsilon^2).$$

Substituting these expressions [and taking into account Eqs. (15) and (16)] into Eq. (3) and rejecting small quantities of high orders, we obtain the following linearized boundary conditions at $x_2 = 0$:

$$\sigma_{12} + \varepsilon f(x_1)\sigma_{12,2} + \varepsilon f'(x_1)(\sigma_{22} - \sigma_{11}) = 0,$$

$$\sigma_{22} + \varepsilon f(x_1)\sigma_{22,2} - 2\varepsilon f'(x_1)\sigma_{12} = -i\omega\rho_*(\varphi + \varepsilon f(x_1)\varphi_{,2}), \quad (17)$$

$$\varphi_{,2} + \varepsilon f(x_1)\varphi_{,22} - \varepsilon f'(x_1)\varphi_{,1} = -i\omega[u_2 + \varepsilon f(x_1)u_{2,2} - \varepsilon f'(x_1)u_1].$$

The solution of the problem is sought in the form of a series with respect to the powers of ε:

$$u_i = u_i^0 + \varepsilon u_i^1 + \ldots, \qquad \varphi = \varphi^0 + \varepsilon\varphi^1 + \ldots\ . \quad (18)$$

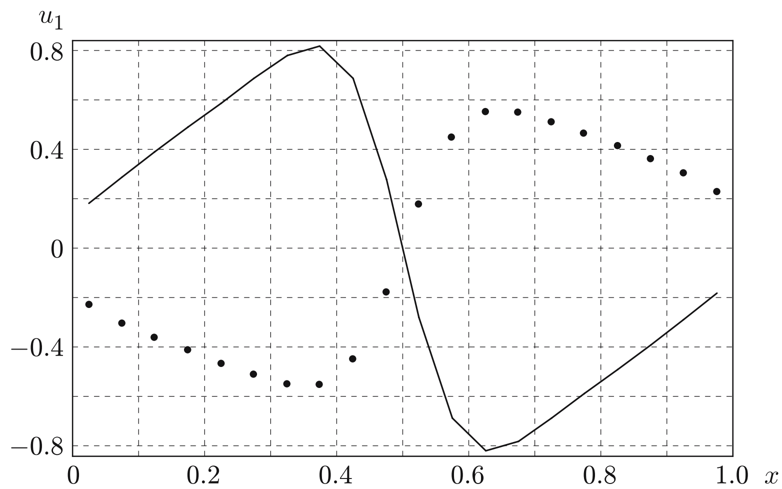

Fig. 1. Results of the numerical solution of the system of integral equations (11)–(14) at $\varkappa_2 = 2$, $\varepsilon = 0.2$, and $f(x_1) = \sin(\pi x_1)$: the solid curve and the points refer to the real and imaginary parts of the horizontal displacement, respectively.

We substitute expansions (17) and (18) into the initial equations (1) and the boundary conditions (2) and (3) and equate the coefficients at identical powers of ε. For u_j^i and φ^i, we obtain the homogeneous Lamé equations and the Helmholtz equation:

$$(1-2\nu)^{-1}u^i_{j,jk} + \Delta u^i_k + \varkappa_2^2 u^i_k = 0, \qquad \Delta\varphi^i + \varkappa_0^2\varphi^i = 0.$$

The following boundary conditions are satisfied:

— for u_i^0 and φ^0 (zeroth approximation),

$$x_2 = 1: \quad \sigma^0_{12} = 0, \quad \sigma^0_{22} = q(x_1),$$
$$x_2 = 0: \quad \sigma^0_{12} = 0, \quad \sigma^0_{22} = -i\omega\rho_*\varphi^0, \quad \varphi^0_{,2} = -i\omega u^0_2;$$

— for u_i^1 and φ^1 (first approximation),

$$x_2 = 1: \quad \sigma^1_{12} = 0, \quad \sigma^1_{22} = 0,$$
$$x_2 = 0: \quad \sigma^1_{12} = -f(x_1)\sigma^0_{12,2} - f'(x_1)(\sigma^0_{22} - \sigma^0_{11}),$$
$$\sigma^1_{22} + i\omega\rho_*\varphi^1 = -f(x_1)(\sigma^0_{22,2} + i\omega\rho_*\varphi^0_{,2}),$$
$$\varphi^1_{,2} + i\omega u^1_2 = -f(x_1)(\varphi^0_{,22} + i\omega u^0_{2,2}) + f'(x_1)(\varphi^0_{,1} + i\omega u^0_1).$$

Both problems are solved with the use of the Fourier transform. The solution obtained in the zeroth approximation coincides with the "reference" solution u_i^{et}.

4. Numerical Solution of the Boundary Equations. System (11)–(14) can be studied only numerically, by the boundary element method [1]. Let us divide the segment $[a, b]$ into N segments $[x_i, x_{i+1}]$ [$i = \overline{0, N-1}$, $x_i = a + hi$, and $h = (b-a)/N$]. The curve Γ_f is approximated by a broken line consisting of straight-line segments (elements) l_i with the ends at the points $l_{i+N} = [(x_i, f(x_i)), (x_{i+1}, f(x_{i+1}))]$. Let the functions u_i be constant on the segments l_i. Points ξ_i located in the middles of the segments l_i are chosen as nodes where the integral equations (11)–(14) are satisfied. For the nodal unknowns u_i, we obtain a system of linear algebraic equations. The coefficients of this system are double integrals, but they can be expressed via single integrals by means of explicit integration over the element.

The calculations are performed for a water–steel layered system. Figure 1 shows the results of the numerical solution of the system of integral equations (11)–(14) at $\varkappa_2 = 2$, $\varepsilon = 0.2$, and $f(x_1) = \sin(\pi x_1)$. After finding the unknowns in the nodes, we can calculate the wave fields on the upper boundary of the layer in accordance with presentation (8). The calculations can also be performed by the method of linearization. In addition, in finding the wave field on the surface with a small roughness amplitude, we can directly substitute the "reference"

TABLE 1

Values of the Parameter δ_1^{I} (in percent)
for Different Roughness Amplitudes and Wavenumbers

ε	$\varkappa_2 = 1$		$\varkappa_2 = 2$	
	$N = 20$	$N = 40$	$N = 20$	$N = 40$
0.1	37.43	38.67	97.17	82.14
0.01	2.43	2.66	3.76	3.67
0.001	3.09	2.53	0.98	0.18
0.0001	3.53	2.60	1.12	0.37

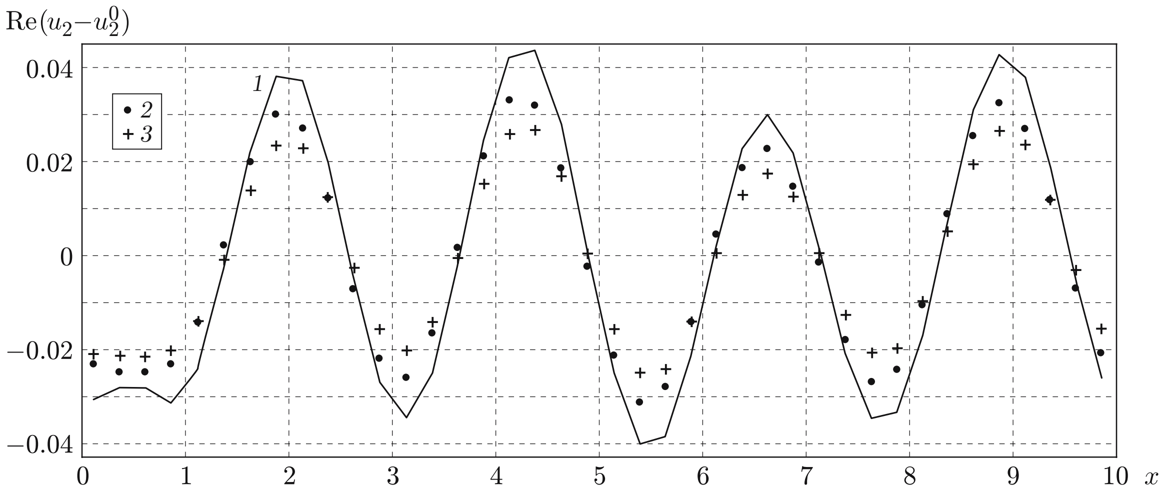

Fig. 2. Dependence of $\mathrm{Re}\,(u_2 - u_2^0)$ on x at $\varkappa_2 = 2$, $\varepsilon = 0.1$, and $f(x_1) = \sin(\pi x_1)$: curve 1 is the solution obtained by the boundary element method; points 2 and 3 are the solutions obtained by the method of linearization and by the Born approximation, respectively.

solution u_i^0 into presentation (8) (this approach is called the Born approximation). Figure 2 shows the dependence of $\mathrm{Re}\,(u_2 - u_2^0)$ on x at $\varkappa_2 = 2$, $\varepsilon = 0.1$, and $f(x_1) = \sin(\pi x_1)$.

Let us denote the wave field found by Eq. (8) by g_i^{I} and the field found by the method of linearization by g_i^{II}. We introduce the quantities

$$\delta_i^{\mathrm{I}} = \frac{\|g_i^{\mathrm{I}} - g_i^{\mathrm{II}}\|}{\|g_i^{\mathrm{I}}\|} \cdot 100\ \%$$

and consider their dependence on the roughness amplitude. The values of the parameter δ_1^{I} for different values of the roughness amplitude and wavenumber are summarized in Table 1.

5. Solution of the Inverse Problem. Let us consider an inverse problem of determining the shape of the boundary from one of the displacement vector components (u_1 or u_2) on the upper surface of the layer, which is known on the segment $[c, d]$. Let us also assume that the "reference" field of displacements $\bar{u}^0$ on the layer surface, which corresponds to the solution of the problem for a layer with a smooth lower surface, is known. Then, we obtain

$$u_i(0, x_1) = u_i^0(0, x_1) + \varepsilon u_i^0(0, x_1) + \ldots$$

and, consequently,

$$\varepsilon u_i^0(0, x_1) = u_1(0, x_1) - u_i^0(0, x_1).$$

Let the function f differ from zero only on the segment $[a, b]$, be continuously differentiable on this segment, and $f(a) = f(b) = 0$. Then, using the presentation for the solution in the first approximation, we obtain

$$u_i^1\Big|_{x_2=1} = \int\limits_a^b f(x_1')K_i^1(x_1, x_1')\,dx_1', \qquad x \in [c, d], \tag{19}$$

where the function $K_i^1(x_1, x_1')$ is rather complicated and is not explicitly expressed.

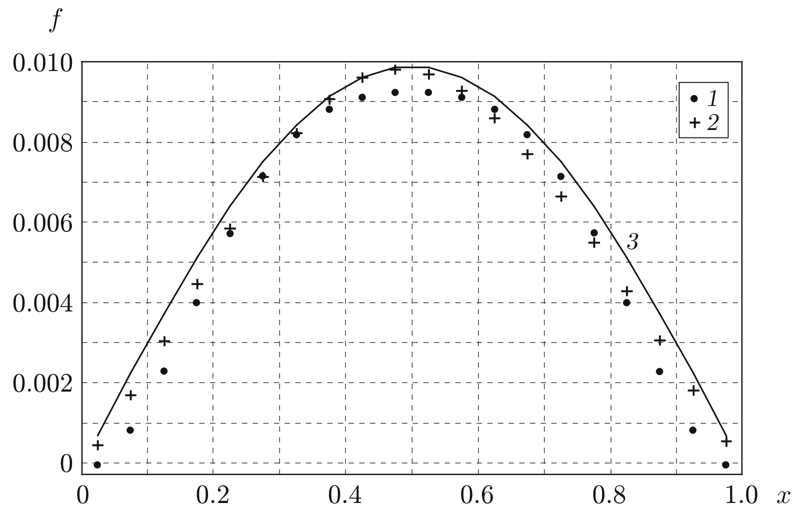

Fig. 3. Results calculated by Eq. (19) at $\varkappa_2 = 4$ and $\varepsilon = 0.01$: points 1 and 2 show the initial roughness shape and the roughness shape reconstructed from the real part of the horizontal displacement on the segment $[2, 4]$; curve 3 is the roughness shape reconstructed from the real part of the horizontal displacement on the segment $[0, 1]$.

Equality (19) is an integral Fredholm equation with a smooth kernel with respect to the function $f(x_1)$. It is known that the procedure of constructing the solution of such an equation is ill-posed and requires regularization [3, 4]. Tikhonov's method of regularization is used in the present work for constructing a regularized solution.

Results of a numerical experiment on reconstructing the roughness shape are presented below. Numerical experiments were performed as follows. First, the system of integral equations (11)–(14) was solved with a known roughness shape to find the unknown displacements on the rough segment of the boundary. Then, Somigliana's relations (8) were applied to the found displacements to determine the fields of displacements on the segment $[c, d]$ of the upper surface of the layer. These displacements were used as the initial data for solving the inverse problem [in the linearized formulation, for solving the integral equation (19)]. The system of linear algebraic equations was solved by Voevodin's method with automatic selection of the regularization parameter, based on the generalized residue criterion.

Figure 3 shows the results calculated by Eq. (19) at $\varkappa_2 = 4$ ($\omega \approx 8$ kHz) and $\varepsilon = 0.01$.

Conclusions. An inverse geometric problem of determining the shape of the rough segment of the interface between the elastic layer and the liquid is reduced by the method of linearization to solving the integral Fredholm equation of the first kind with a smooth kernel. Numerical experiments show that this approach is only effective if the roughness amplitude is small and does not exceed 0.1 of the layer width. At greater roughness amplitudes, iterative schemes of solving inverse problems should be used [9]. The approach proposed can be used for detecting roughness elements in pipelines and reservoirs.

REFERENCES

1. D. Colton and R. Kress, *Integral Equation Methods in Scattering Theory*, Wiley, New York (1983).
2. A. O. Vatul'yan and S. A. Korenskii, "Method of linearization in geometric inverse problems of the elasticity theory," *Prikl. Mat. Mekh.*, **61**, No. 4, 639–646 (1997).
3. A. O. Vatul'yan and P. S. Uglich, "Determining the shape of the rough interface between elastic and liquid media," *Izv. Vyssh. Uchebn. Zaved., Sev-Kavk. Region*, Special issue: *Mat. Model.* (2001), pp. 44–46.
4. A. D. Zakharenko, "Scattering of sound on small compact inhomogeneities in a sea waveguide," *Akust. Zh.*, **46**, No. 2, 200–204 (2000).
5. A. D. Zakharenko, "Scattering of sound on small compact inhomogeneities in a sea waveguide: inverse problem," *Akust. Zh.*, **48**, No. 2, 200–204 (2002).

6. A. A. Goryunov and A. V. Saskovets, *Inverse Problems of Scattering in Acoustics* [in Russian], Izd. Mosk. Univ., Moscow (1989).
7. A. N. Tikhonov and V. Ya. Arsenin, *Methods for Solving Ill-Posed Problems* [in Russian], Nauka, Moscow (1986).
8. I. I. Vorovich and V. A. Babeshko, *Dynamic Mixed Problems of the Theory of Elasticity for Nonclassical Domains* [in Russian], Nauka, Moscow (1979).
9. O. M. Alifanov, E. A. Artyukhin, and S. V. Rumyantsev, *Extreme Methods of Solving Ill-Posed Problems* [in Russian], Nauka, Moscow (1988).